Macro stereography using image magnification

If a macro lens is calibrated for distance then the "1 in 15" rule is easy to apply. However if the lens is calibrated in terms of magnification (or the stereographer prefers using magnification rather than near distance), then it is possible to write the "1 in 15" rule for near distance in terms on macro lens magnification.

The depth of field Δ from the object distance $S_{\rm O}$ to the near point $D_{\rm N}$ is

$$\Delta = S_0 - D_N$$

Since the near point distance is related to the hyperfocal distance D_{H} via the relation

$$\mathsf{D}_{\mathsf{N}} = \frac{\mathsf{D}_{\mathsf{H}}\mathsf{S}_{\mathsf{O}}}{\mathsf{D}_{\mathsf{H}} + \left(\mathsf{S}_{\mathsf{O}} - \mathsf{F}_{\mathsf{C}}\right)}$$

then this depth of field can be written as

$$\Delta = S_{0} - \frac{D_{H}S_{0}}{D_{H} + (S_{0} - F_{C})}$$
$$= S_{0} \left(1 - \frac{1}{1 + \left(\frac{S_{0} - F_{C}}{D_{H}}\right)} \right)$$

Using the thin lens equation it can shown that

$$S_{O} - F_{C} = \frac{F_{C}}{M}$$

where M is the magnification of the lens when focussed at the object distance. Substituting this into the previous expression and simplifying yields the result

$$\Delta = \frac{\mathsf{S}_{\mathsf{O}} \,\mathsf{F}_{\mathsf{C}}}{\mathsf{M}\mathsf{D}_{\mathsf{H}} + \mathsf{F}_{\mathsf{C}}}$$

Using the definition of hyperfocal distance and the fact that $MD_{H} \gg F_{C}$ and so $MD_{H} + F_{C} \approx MD_{H}$ yields,

$$\Delta = \frac{S_{O}CN}{MF_{C}}$$

Similarly if the depth of field behind the object distance were to be found from $\Delta = D_F - S_O$, then using the definition of far point, namely

$$\mathsf{D}_{\mathsf{F}} = \frac{\mathsf{D}_{\mathsf{H}} \mathsf{S}_{\mathsf{O}}}{\mathsf{D}_{\mathsf{H}} - (\mathsf{S}_{\mathsf{O}} - \mathsf{F}_{\mathsf{C}})}$$

would give a similar expression,

$$\Delta = \frac{\mathsf{S}_{\mathsf{O}} \: \mathsf{F}_{\mathsf{C}}}{\mathsf{M}\mathsf{D}_{\mathsf{H}} - \mathsf{F}_{\mathsf{C}}}$$

with the only difference being the minus sign in the denominator. Again, $MD_H \gg F_C$ and $MD_H - F_C \approx MD_H$ and the depth of field is

$$\Delta = \frac{S_{O}CN}{MF_{C}}$$

That is, the depth of field in front of the object is the same as the depth of field behind it and the total depth of field is simply 2Δ . In addition, the depth of field does not depend on the lens focal length and, since $S_0/F_c = (M+1)/M$, can also be written as

$$2\Delta = \frac{2CN(M+1)}{M^2}$$

Hence the near point distance (and thus the stereo base as well) can be written in terms of the lens magnification.

$$D_{N} = S_{O} - \Delta = S_{O} - \frac{S_{O}CN}{MF_{C}}$$
$$= \frac{S_{O}}{F_{C}} \left(\frac{MF_{C} - CN}{M}\right)$$

From the thin lens equation, as earlier, $S_0/F_c = (M+1)/M$ and so

$$\mathsf{D}_{\mathsf{N}} = \left(\frac{\mathsf{M}+1}{\mathsf{M}^2}\right) \left(\mathsf{MF}_{\mathsf{C}} - \mathsf{CN}\right)$$

However MF_C » CN and so MF_C – CN \approx $MF_C.$ Hence the near distance can be written accurately as

$$D_{N} = F_{C}\left(\frac{M+1}{M}\right)$$

Thus the stereo base can be written in terms of magnification M as

$$\mathsf{B} = \frac{\mathsf{Fc}}{15} \left(1 + \frac{1}{\mathsf{M}} \right)$$

Note that in the special case of lifesize (M = 1) macro stereography, the stereo base is simply B = Fc / 7.5.

A graph of stereo base versus magnification for a 50mm focal length lens is displayed below, providing stereo base information at a glance.

