THE DI MARZIO EQUATION

The problem of the proper stereo base relevant to almost any situation was first addressed by John Bercovitz and others. They used a simple geometrical approach to derive an expression for this stereo base B in terms of the on-film deviation, the object distance So, the focal length of the camera lens Fc and the near Dn and far Df points required to be in acceptably sharp focus.

The relationship is known as the Bercovitz equation, and although it is a very general expression, it can be difficult to use for the general stereographer and needs to be re-evaluated for every scene being photographed. Moreover, the stereo base calculated using the Bercovitz equation has to be corrected in order to cater for any mismatch between the focal length of the camera lens and that of the stereo viewer. This correction can be quite complicated as it depends not only on the focal lengths of the camera and stereo viewer lenses, but also on the position of the final image.

However, there is a much simpler means by which the stereo base can be determined. Instead of treating the near and far points as independent variables, they can be related to one another through the f/number N. For any given near point distance Dn, the f/number determines the value for the acceptably sharp far point distance Df, and vice versa.

This allows an enormous simplification of the Bercovitz equation into a form that is very easy to use. The final result for stereo base is exact and is known as the Di Marzio equation. It can be written as

$$B = \frac{100Fc}{3N}$$

where B is the stereo base, Fc is the focal length of the camera lens and N is the smallest f/number (largest aperture) capable of providing a depth of field ranging from the near point Dn up to the far point Df.

This equation holds true for all scenes in which $Df \geq 2Dn$ and is remarkably easy to use. Shallow objects in which $Df < 2Dn$ can also be handled easily and this will be discussed later.

Example of stereography with a 50mm focal length camera

For a 50mm focal length camera lens, the Di Marzio equation for the stereo base is just

$$B = \frac{100x50}{3N} = \frac{1667}{N}$$

In addition, there are no calculations at all required in the field. The stereo base can be found by using a look-up table of common f/numbers. A typical look-up table for a 50mm lens is displayed below.

f/number N	B (mm)
22	76
19.6	85
16	104
13.5	123
11	152
9.8	170
8	208
6.9	242
5.6	298

All that needs to be done is to focus the camera on the near point Dn in the scene and read this distance from the lens rangefinder. Similarly by focussing on the far point, the far distance Df can also be found.

It is then a simple matter to use the lens depth of field scale to determine the smallest f/number N that can accommodate this range of distances. For instance, if this smallest f/number is f/11, then the stereo base is 152mm.

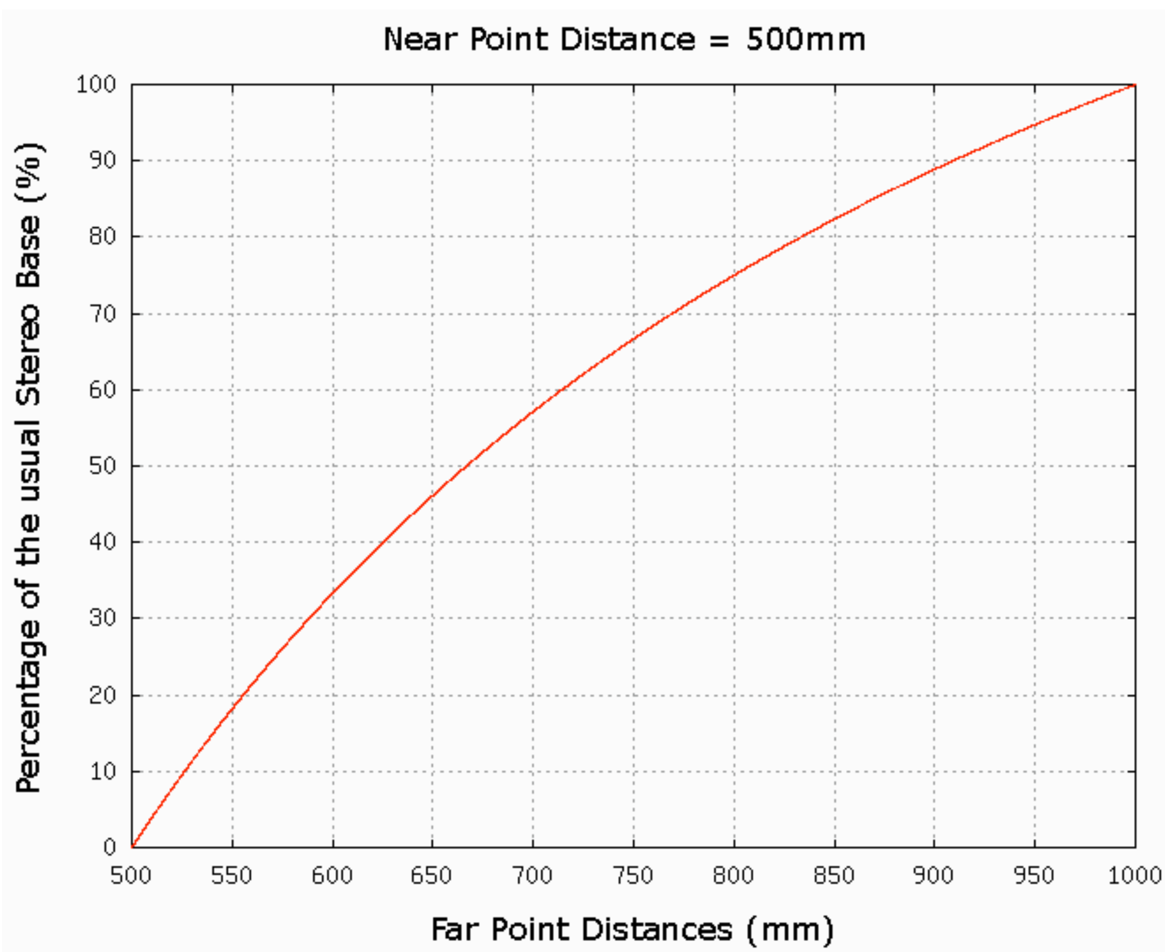
SPECIAL CASES

Hypostereography of Shallow Objects

If the Bercovitz equation is used to calculate stereo base for situations in which Df < 2Dn, the image depth produced is too great.

Michael K. Davis and others have suggested that when Df < 2Dn, one should simply determine the near point distance Dn and set the far point at Df = 2Dn. Since Df is actually less than 2Dn this yields a smaller value for the stereo base than that of the Bercovitz equation thereby properly allowing for diminishing depth of the subject.

This is an excellent modification as it permits the stereo base to be a smoothly decreasing percentage of the Bercovitz base. An example of this variation is displayed below for a near point distance of 500mm.



Not only does this deliberate tapering of the stereo base avoid exaggerated depth and allow a proper depth rendition in shallow subjects, but it also permits a trivial calculation for the stereo base.

The problem with the Bercovitz equation for shallow objects arises from the denominator in his equation. There is a Df – Dn term which makes the base too large for shallow objects. As you endeavour to photograph shallower and shallower objects, this term in the denominator becomes smaller and smaller, thereby making the stereo base unrealistically larger and larger.

With the Bercovitz equation, as the denominator Df – Dn approaches zero, the base approaches infinity — not a reasonable scenario. So the beauty of the Davis modification is twofold. It allows a smooth transition from deep to shallow subjects and it completely avoids unrealistically large stereo bases for very shallow subjects.

In fact, by using the Davis modification and setting Df = 2Dn an amazingly simple expression for stereo base emerges which depends only on the near point distance, viz.

$$B = \frac{Dn}{15}$$

In addition, it is possible to recast this equation in terms of the focal length and magnification of the lens, namely

$$B = \frac{Fc}{15}\left(1 + \frac{1}{M}\right)$$

Note that in the special case of lifesize (M = 1) macro stereography, the stereo base is simply B = Fc/7.5.

SPECIAL CASES

Hyperstereography of Distant Subjects

Scenes with Df > 2Dn do not present a problem unless Dn itself is large. In these cases it becomes difficult if not impossible to use a depth of field scale to find the f/number N for determining the stereo base. However, there is a simple way around this problem.

If Dn is large (several metres or more) and Df = mDn where m ≥ 2, then the Bercovitz equation can be simplified yet again, yielding

$$B = \left(\frac{m}{m-1}\right)\frac{Dn}{30}$$

Note that when m is large, that is, when the far point distance is much greater than the near point, such as in landscape stereography, then

$$\left(\frac{m}{m-1}\right) \rightarrow 1 \quad \text{and} \quad B \rightarrow \frac{Dn}{30}$$

which is the familiar "one in thirty" rule.

The "one in thirty" rule therefore is valid only when Df is much larger than Dn. Clearly such a rule is inadequate for shallow objects which are more realistically described by a "one in fifteen" rule.

The modified one in thirty rule can also be used for smaller Dn, and involves only one simple calculation.

Subtleties and Technicalities of the Di Marzio Equation

- The f/number N in the Di Marzio equation is the smallest f/number capable of covering the desired depth of field of the scene. It is this smallest value that is used to ascertain the stereo base.

- The smallest f/number N is a computational tool employed to find the stereo base. It does not necessarily have to be the f/number that is set on the camera lens. The stereographer is free to use whatever f/number they wish on their lens.

- Since the base B depends on the (smallest) f/number N, then any combination of near and far points producing a depth of field compatible with N will require the same stereo base. There is no need to redetermine the stereo base in these cases.

- The most fundamental form of the Di Marzio equation is

$$B = \frac{Dh}{60}$$

  where Dh is the hyperfocal distance namely, the object distance at which everything between half that distance and infinity is in acceptably sharp focus.

  This is an extraordinarily simple and powerful equation which provides the foundation for handling essentially any aspect of stereography. It is from this fundamental form of the Di Marzio equation that other forms are derived.

- In the Bercovitz equation, the on-film deviation (ofd) can be set to any value. Generally the ofd is given a value of Fc/30 where Fc is the camera lens focal length. In this way the ofd provides a parallax angle of 1.9°, an angle which provides pleasing image depth for most people.

But an ofd of Fc/30 only provides a parallax angle of 1.9° when the object distance is much greater than the focal length of the camera, that is, when the lens is a distance Fc from the film plane. However for close up and, in particular, macro stereography, the lens is significantly further away from the film plane and so an ofd of Fc/30 will be too small, leading to somewhat shallow images.

In all forms of the Di Marzio equation an on-film deviation of Si/30 is used, where Si is the image distance (the distance between the camera lens and the film plane). This guarantees that a parallax angle of 1.9° is maintained irrespective of the type of stereography undertaken.

► When freeviewing, the optimum viewing distance is MSi where M is the image enlargement factor and Si is the image distance as before. In general stereography, Si ≈ Fc and so this optimum freeviewing distance is MFc.

Corrections for a Stereo Viewer

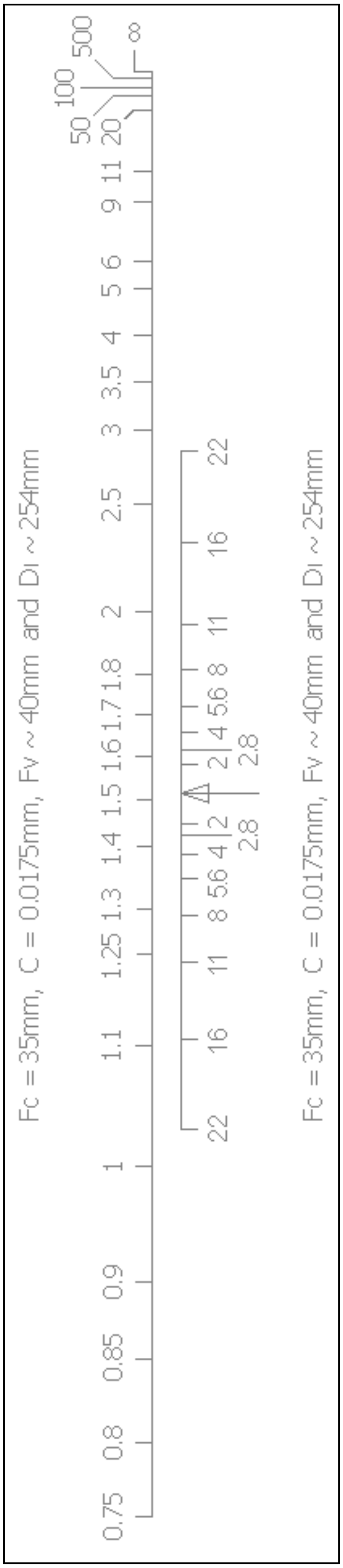
The Bercovitz equation enables the evaluation of the stereo base required to provide a specified on-film deviation. However it does not account for arbitrary freeviewing distances nor, in particular, for disparities between the taking (camera) and viewing optical systems.

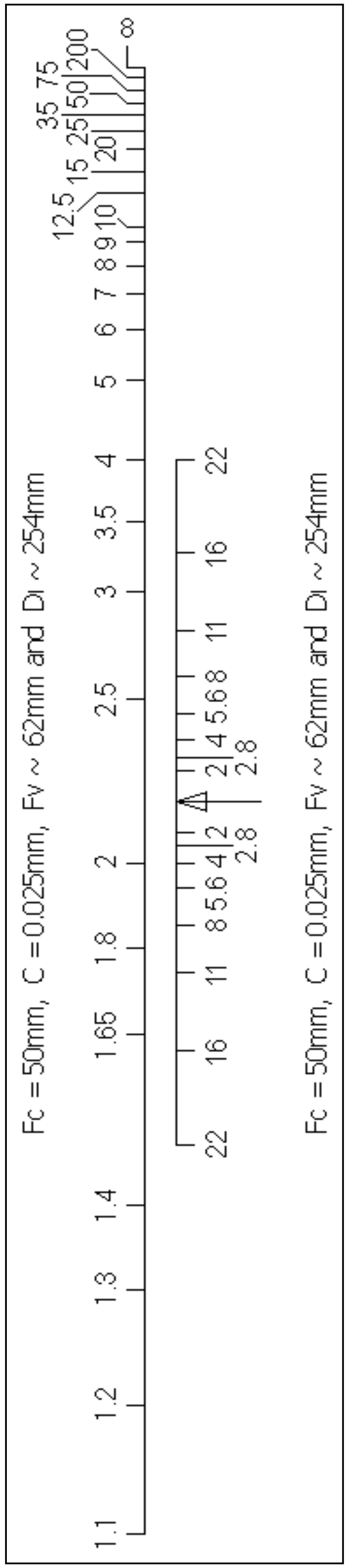
► One of the most powerful aspects of the Di Marzio equation is that it can automatically cater for differences in the focal lengths of the camera and stereo viewer lenses. A correction factor for this focal length mismatch can be encapsulated within the f/number itself.

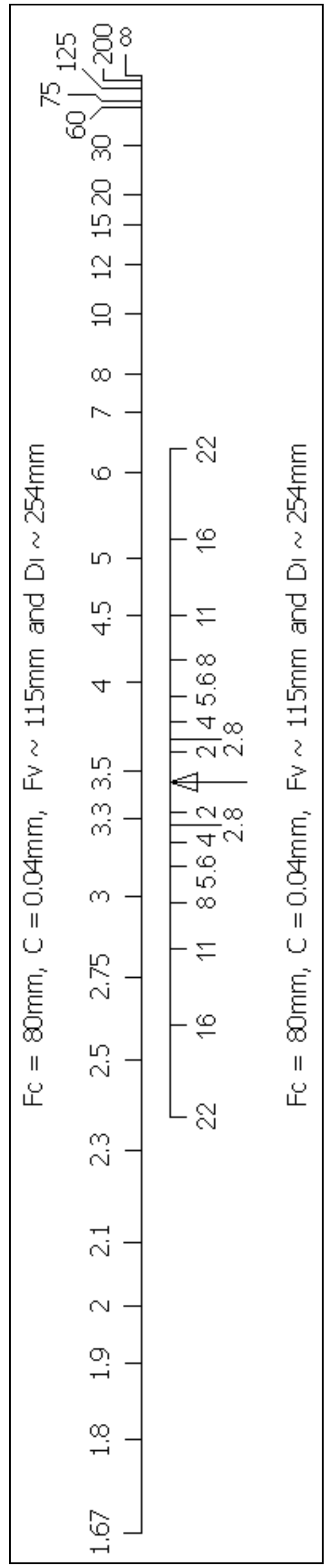
Three examples of depth of field scales specifically generated for images to be captured with a 35mm, 50mm and 80mm camera lens are displayed below.

The reason each scale has its own particular stereo viewer focal length and final image distance is because this combination provides the same stereo base for both freeviewing and for use with this stereo viewer.

In fact it can also be used for determination of depth of field in more conventional 2D photography. Consequently, each depth of field scale serves three very useful purposes.


Fc = 35mm, C = 0.0175mm, Fv ~ 40mm and DI ~ 254mm
Fc = 35mm, C = 0.0175mm, Fv ~ 40mm and DI ~ 254mm


Fc = 50mm, C = 0.025mm, Fv ~ 62mm and DI ~ 254mm
Fc = 50mm, C = 0.025mm, Fv ~ 62mm and DI ~ 254mm


Fc = 80mm, C = 0.04mm, Fv ~ 115mm and DI ~ 254mm
Fc = 80mm, C = 0.04mm, Fv ~ 115mm and DI ~ 254mm

For example, suppose a 50mm focal length camera is used to photograph a scene with Dn = 2.5m and Df ≈ 12m. The depth of field scale for Fc = 50mm indicates that this can achieved with a smallest f/number of about f/16. Therefore a stereo base of 104mm will do the job nicely (see earlier stereo base table for the 50mm lens).

Moreover, the final image will show realistic depth whether it is freeviewed from the optimum distance or viewed using a 62mm focal length viewer with a final image placed some 250mm behind the lenses.

- Although the characteristics of the depth of field scale depends on the diameter of the circle of confusion C, the stereo base does not. Specifically, the Di Marzio equation for stereo base contains a factor CN. However N is inversely proportional to C. So if you double the value of C, the value for N halves. That is, irrespective of the value for C, the factor CN remains the same. Consequently a general value of C = Fc/2000 is used in the Di Marzio equation and for generating depth of field scales.

- In general, a depth of field scale appropriate for both a stereo viewer and for freeviewing can be generated provided the stereo viewer has lenses with a focal length Fv given by

$$\frac{1}{Fv} = \frac{1}{Fc} - \frac{1}{Di}$$

where Fc is the camera focal length and Di is the distance of the final (viewed) image behind the stereo viewer lenses. Sometimes it may be better to find a Di that will match a commonly available viewer lens focal length.

► The correction to the (freeviewing) stereo base required to match viewer and camera lenses can be applied to any of the forms of the Di Marzio equation. In macro stereography, for example, the stereo base Bv for use in conjunction with a stereo viewer can be written as

$$Bv(Di = \infty) = \frac{Fv}{15M}\left(1 + \frac{1}{M}\right)$$

where Fv is the stereo viewer lens focal length and M is the magnification of the camera lens. The final viewed image is assumed to be very distant ($Di = \infty$). For lifesize ($M=1$) macro stereography, the base is trivial, namely $Bv = Fv/7.5$.